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LETTER TO THE EDITOR

Empirical evidence for a boundary-induced nonequilibrium phase transition

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Abstract
A recently developed theory for boundary-induced phenomena in nonequilibrium systems predicts the existence of various steady-state phase transitions induced by the motion of a shock wave. We provide direct empirical evidence that a phase transition between a free flow and a congested phase occurring in traffic flow on highways in the vicinity of on- and off-ramps can be interpreted as an example of such a boundary-induced phase transition of first order. We analyse the empirical traffic data and give a theoretical interpretation of the transition in terms of the macroscopic current. Additionally we support the theory with computer simulations of the Nagel–Schreckenberg model of vehicular traffic on a road segment which also exhibits the expected second-order transition. Our results suggest ways to predict and to some extent to optimize the capacity of a general traffic network.

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One-dimensional physical systems with short-ranged interactions in thermal equilibrium do not exhibit phase transitions. However, this is no longer true if the action of external forces sets up a steady mass transport and drives the system out of equilibrium. Then boundary conditions, usually insignificant for an equilibrium system, can induce nonequilibrium phase transitions in the stationary states of both deterministic and noisy driven complex systems, including biological and sociological mechanisms involving many interacting agents. Despite the importance of this phenomenon and a number of theoretical studies [1–4], it has never been observed directly. So far, only indirect experimental evidence for a boundary-induced phase transition exists in older studies of the stochastic kinetics of biopolymerization on nucleic acid templates [5, 6].

In the present work we report the first direct observation of a boundary-induced steady-state phase transition of first order which occurs in traffic flow. Statistical analysis of traffic data sets taken on a motorway near Cologne reveals transitions from stationary free-flow traffic to stationary congested traffic, characterized by a discontinuous reduction of the average speed [7] and caused by a boundary effect, viz. the presence of an on-ramp. We show that these data find unambiguous interpretation in the framework of nonequilibrium phase transition theory for stationary states as applied to a road segment between the on- and off-ramps. The theory predicts also a second-order phase transition, the existence of which we demonstrate by Monte Carlo simulation. To avoid confusion we stress that we study self-organized stationary behaviour (observed in averaging over sufficiently long time intervals, see below) as a function of external control parameters (the activity on the on-ramp and the inflow into the road segment) rather than effects triggered by a moving perturbation [8, 9].

Vehicular traffic on a motorway is controlled by a mixture of bulk and boundary effects caused by on- and off-ramps, varying number of lanes, speed limits, weather conditions, etc. The fundamental characteristic of the bulk motion is the stationary flow–density diagram, i.e. the fundamental diagram, which incorporates the collective effects of individual drivers behaviour such as trying to maintain an optimal high speed while taking safety precautions. The qualitative shape of the flow–density diagram $j(\rho)$ is largely independent of the precise details of the road and hence amenable to numerical analysis using either stochastic lattice gas models or partial differential equations [10, 11]. A decisive conceptual feature of our theoretical picture is its *derivation* from the stochastic microdynamics in the lattice gas framework [4, 12] from which we conclude that only the qualitative features of the flow–density diagram, but not the microscopic details of traffic flow, determine the stationary phase diagram. This basic strategy and its ramifications (in particular, the incorporation of fluctuations due to noise and the stochastic treatment of on/off-ramps) is in contrast to the coarse-grained, gas-kinetic approach of [8] which entails a deterministic and more phenomenological description of traffic flow phenomena.

We illustrate our theoretical approach by using a now well established lattice gas model for traffic flow, the cellular automaton model of Nagel and Schreckenberg [13] (NaSch model). The flow–density relation for the NaSch model, figure 1, is in agreement with measurements of the flow j taken with the help of detectors on the motorway A1 near Cologne which show a maximum of about 2000 vehicles h^{-1} at a density of about $\rho^* = 20$ vehicles km^{-1} $lane^{-1}$ [7]. At densities below ρ^* one observes free flow, while for larger densities one observes congested traffic.

In addition to the density dependence of the flow, two important characteristics are derived directly from the fundamental diagram: the shock velocity of a 'domain wall' between two stationary regions of densities ρ^-, ρ^+,

$$v_{\text{shock}} = \frac{j(\rho^+) - j(\rho^-)}{\rho^+ - \rho^-}, \tag{1}$$

obtained from mass conservation, and the collective velocity,

$$v_{\text{c}} = \frac{\partial j(\rho)}{\partial \rho} \tag{2}$$

which is the *mean* velocity of the centre of mass of a local perturbation spreading slowly out in a homogeneous, stationary background of density ρ. Both velocities are readily observed in real traffic. The collective velocity v_c describes the upstream movement of a local, compact jam. In the density range 25–130 cars km^{-1}, v_c ranges from approximately -10 km h^{-1} to -20 km h^{-1} (figure 1) which has to be compared with the empirically observed value $v \approx -15$ km h^{-1} [13, 14]. The shock velocity is the mean velocity of the (fluctuating) position of the upstream front of an extended, stable traffic jam. The formation of a stable shock is usually a boundary-driven process, caused by a 'bottleneck' on a road. Bottlenecks on a highway may

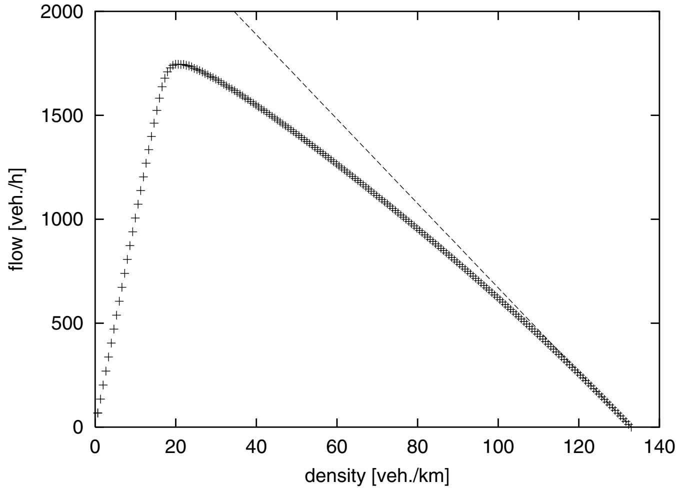

Figure 1. Fundamental diagram (flow–density relation) as modelled by the NaSch model with $v_{\max} = 4$, randomization parameter $p = 0.25$, time step $= 1.0$ s, lattice spacing $= 7.5$ m. The system has 3200 sites, and the flux is averaged over 10^6 lattice updates. The broken and full lines indicate the slope which defines the collective velocity of spontaneous local traffic jams and the shock velocity, respectively.

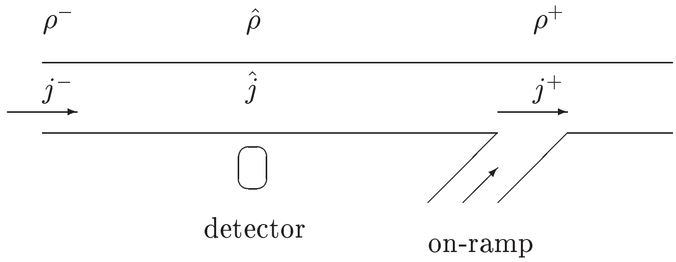

Figure 2. Schematic road design of a highway with an on-ramp where cars enter the road. The arrows indicate the direction of the flow. The detector measures the local bulk density $\hat{\rho}$ and bulk flow $\hat{j}$.

arise from a reduction in the number of lanes or from on-ramps where additional cars enter a road [15, 16].

The experimental data considered here (see figure 2 for the relevant part of the highway) show boundary effects caused by the presence of an on-ramp. Far upstream from the on-ramp, free flow of vehicles with density ρ^- and flow $j^- \equiv j(\rho^-)$ is maintained. Just before the on-ramp, the vehicle density is ρ^+ with corresponding flow $j^+ \equiv j(\rho^+)$. Note that no experimental data are available for ρ^-, j^- and ρ^+, j^+ as well as the activity of the ramp. The only data come from a detector located upstream from the on-ramp[5] which measures a traffic density $\hat{\rho}$ and the corresponding flow $\hat{j}$.

Next the effects of the on-ramp are considered. Cars entering the motorway cause the mainstream of vehicles to slow down locally. Therefore, the vehicle density just before the

[5] The distance between the detector and the on-ramp should be large enough, so that the on-ramp fluctuations are not measured directly. In our case, the detector is located approximately 1 km upstream from on-ramp.

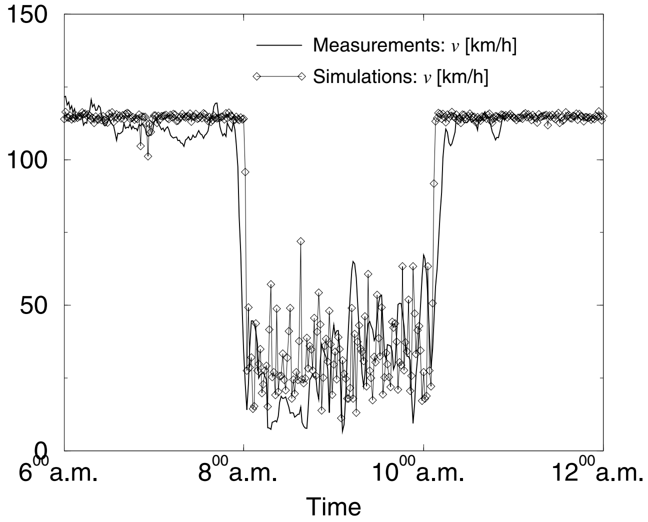

Figure 3. Time series of the velocity. Each data point represents an one-minute average of the speed. Shown are empirical data (from [7]) in comparison with the results of computer simulations of a simplified model (see text).

on-ramp increases to $\rho^+ > \rho^-$. Then a shock, formed at the on-ramp, will propagate with *mean* velocity $v_{\rm shock}$ (see (1)). Depending on the sign of $v_{\rm shock}$, two scenarios are possible:

(1) $v_{\rm shock} > 0$ (i.e. $j^+ > j^-$). In this case the shock propagates (on average) downstream towards the on-ramp. Only by fluctuations is a brief upstream motion possible. Therefore the detector will measure a traffic density $\hat{\rho} = \rho^-$ and flow $\hat{\jmath} = j^-$.

(2) $v_{\rm shock} < 0$ (i.e. $j^+ < j^-$). The shock wave starts propagating with the mean velocity $v_{\rm shock}$ upstream, thus expanding the congested traffic region with density ρ^+. The detector will now measure $\hat{\rho} = \rho^+$ and flow $\hat{\jmath} = j^+$.

Let us now discuss the transition between these two scenarios. Suppose one starts with a situation where $j^+ > j^-$ is realized. If now the far-upstream density ρ^- increases it will reach a critical point $\rho_{\rm crit} < \rho^*$ above which $j^- > j^+$; i.e. the free flow upstream j^- prevails over the flow j^+ which the 'bottleneck' (the on-ramp) is able to support. At this point the shock wave velocity $v_{\rm shock}$ will change sign (see (1)) and the shock starts travelling upstream. As a result, the stationary bulk density $\hat{\rho}$ measured by the detector upstream from the on-ramp will change discontinuously from the critical value $\rho_{\rm crit}$ to ρ^+. This marks a nonequilibrium phase transition of first order with respect to the order parameter $\hat{\rho}$. The discontinuous change of $\hat{\rho}$ leads also to an abrupt reduction of the local velocity. Notice that the flow $\hat{\jmath} = j^+$ through the on-ramp (then also measured by the detector) will stay *independent* of the free flow upstream from the congested region j^- as long as the condition $j^- > j^+$ holds.

Empirically this phenomenon can be seen in the traffic data taken from measurements at the detector D1 on the motorway A1 close to Cologne [7]. Figure 3 shows a typical time series of the one-minute velocity averages. One can clearly see the sharp drop in the velocity at about 8 am.

Also the measurements of the flow versus local density, i.e. the fundamental diagram (figure 4), support our interpretation. Two branches can be distinguished. The increasing part corresponds to an almost linear rise of the flow with density in the free-flow regime [14]. In

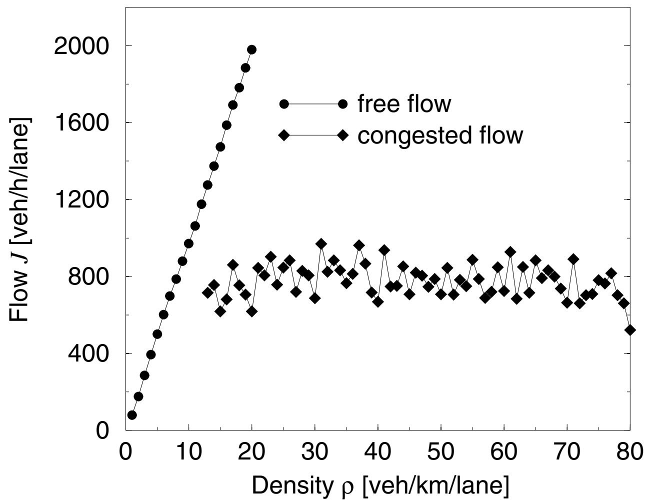

Figure 4. Measurements of the flow versus local density before an on-ramp on the motorway A1 close to Cologne (data from [7]). The detector is located at a distance 1 km upstream from the on-ramp. The data were collected over a period of 12 days.

accordance with our considerations, this part of the flow diagram is not affected by the presence of the on-ramp at all and one measures $\hat{j} = j^-$ which is the actual upstream flow. The second branch are measurements taken during congested traffic hours, the transition period being omitted for better statistics. The transition from free flow to congested traffic is characterized by a discontinuous reduction of the local velocity. However, as predicted above, the flow does not change significantly in the congested regime. In contrast, in local measurements large density fluctuations can be observed. Therefore in this regime the density does not take the constant value ρ^+ as suggested by the argument given above, but varies from 20 to 80 vehicles km^{-1} $lane^{-1}$ (see figure 4).

One should stress here that congested traffic data are usually not easy to interpret, because the traffic conditions (mean inflow and outflow of cars on the on- and off-ramps, and so the bulk mean flow) are changing in time. According to our arguments, in a congested regime the detector measures j^+, *solely* due to the on-ramp activity. Therefore, $j^+(t) = j(\rho^+(t)) < j^-(t)$ must be satisfied. During times of very dense traffic one expects there are always cars ready to enter the motorway at the on-ramp, thus guaranteeing a sufficient and approximately constant on-ramp activity. The measured flow is constant over long periods of time which is in agreement with the notion that the transition is due to a stable traffic jam. Spontaneously emerging and decaying jams would lead to the observation of a non-constant flow.

The use of our approach is not limited to a qualitative explanation of the data. Beyond that it can also be used to calculate the phase diagrams of systems with open boundary conditions for a large class of traffic models. We modelled a section of a road, with on-ramp on the left and with either an off-ramp or an on-ramp on the right, using the NaSch cellular automaton [13]. We modify the basic model by using open boundary conditions with injection of cars at the left boundary (corresponding to in-flow into the road segment) and removal of cars at the right boundary (corresponding to out-flow). Therefore it can also be regarded as a generalization of the asymmetric simple exclusion process (see e.g. [17]) to particles with higher velocity.

During the simulations local measurements of the velocity have been performed analogous to the experimental setup. For comparison the results of the computer simulations have been

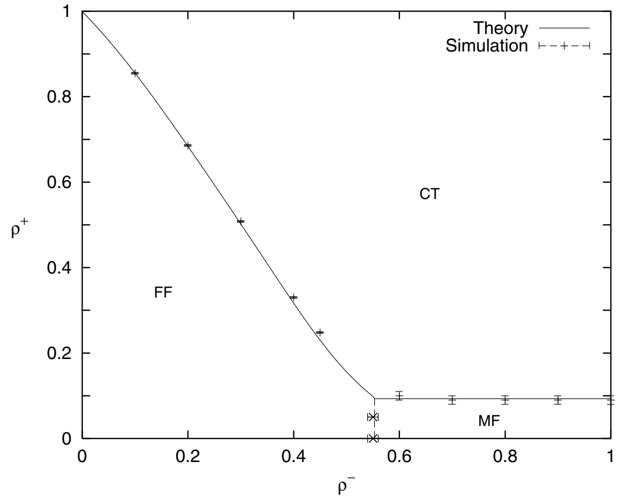

Figure 5. Phase diagram of the NaSch model with open boundaries for $p = 0.25$, $v_{max} = 4$. Cars enter the road from a reservoir of density $\overline{\rho^-}$, inducing the upstream density ρ^- discussed in the text. At the right boundary cars leave the road into a reservoir of density $\overline{\rho^+}$, leading to the on-ramp density ρ^+. The solid (dashed) curve denotes the theoretical prediction for the first-order (second-order) transitions lines obtained from the numerically determined flow–density relation. The points represent phase transition points for the simulated system of size 3200. The phases are: free flow (FF), congested traffic (CT), maximal flow (MF) phase.

included in figure 3. Note that even the quantitive agreement with the empirical data is very good. This has been achieved by using a finer discretization of the model, i.e. the length of the cell is considered as $l = 2.5$ m. The results were obtained for $L = 960$, $p = 0.25$ and $v_{max} = 13$. We kept the input probability $\alpha = 0.65$ constant. Then the free-flow part is obtained using $\beta = 1$ and the congested part using $\beta = 0.55$. The transition was observed at 10 min after we reduced the output probability. The 'detector' was located at the link from site 480 to 481.

Figure 5 shows the full phase diagram of the NaSch model with open boundary conditions. It describes the stationary bulk density $\hat{\rho}$ as a function of the far-upstream in-flow boundary density ρ^- and the effective right boundary density ρ^+. The case of an on-ramp (or shrinking road width, etc) at the right boundary corresponds to the situation discussed above. Here, the density is increased locally to $\rho^+ > \rho^-$. In agreement with the empirical observation, we find a line of first-order transitions from a free-flow (FF) phase with bulk density $\hat{\rho} = \rho^-$ to a congested traffic (CT) phase with $\hat{\rho} = \rho^+$. On this line v_{shock} changes sign.

The case of an off-ramp (or expansion of road space, etc) leads to a local decrease $\rho^+ < \rho^-$ of the density. Here the collective velocity v_c (2) plays a prominent role. As long as v_c is positive (i.e. in the free-flow regime $\rho^- < \rho^*$, see figure 1), perturbations caused by a small increase of the upstream boundary density ρ^- gradually spread into the bulk, rendering $\hat{\rho} = \rho^-$ (FF regime). At $\rho^- = \rho^*$, v_c changes sign[6] and an overfeeding effect occurs: a perturbation from the upstream boundary does not spread into the bulk [2, 4] and therefore further increase of the upstream boundary density does not increase the bulk density. The system enters the maximal flow (MF) phase with constant bulk density $\hat{\rho} = \rho^*$ and flow $j(\rho^*) = j_{max}$. The transition to the MF phase is of second order, because $\hat{\rho}$ changes continuously across the phase transition point.

[6] In this case the upstream entrance to the road itself acts as a 'dynamical' bottleneck with maximal capacity j_{max}.

The existence of a maximal flow phase has not been emphasized in the context of traffic flow up to now. At the same time, it is the most desirable phase, carrying the maximal possible throughput of vehicles j_{max}. For practical purposes our observations may directly be used in order to operate a highway in the optimal regime. The maximal throughput can be observed if both density reservoirs have a higher capacity than the considered stretch of the highway, e.g. a two-lane section between two three-lane sections. In this example the maximal capacity can be achieved by optimizing the lane changes upstream of the lane reduction.

We stress that the stationary phase diagram of figure 5 is generic in the sense that it is determined solely by the macroscopic flow–density relation. The number of lanes of the road, the distribution of individual optimal velocities, speed limits and other details enter only in so far as they determine the exact values characterizing the flow–density relation for that particular road. We also note that throughout the paper we have assumed the external conditions to vary slowly, so that the system has enough time to readjust to its new stationary state. Experimenting with different cellular traffic models in a real time scale shows that the typical time to reach a stationary state in a road segment of about 1.2 km is of the order of 3–5 min, which is reasonably small.

In conclusion, we have shown that traffic data collected on German motorways provide evidence for a boundary-induced nonequilibrium phase transition of first order from the free flowing to the congested phase. The features of this phenomenon are readily understood in terms of the flow–density diagram. The dynamical mechanism leading to this transition is an interplay of shocks and local fluctuations caused by an on-ramp. Full investigation of a cellular automaton model for traffic flow reproduces this phase transition, but also exhibits a richer phase diagram with an interesting maximal flow phase. These results are not only important from the point of view of nonequilibrium physics, but also suggest new mechanisms of traffic control.

We stress that our considerations apply to arbitrary models of traffic flow with a single-valued current–density relation $j(\rho)$, see e.g. [12]. There are experimental indications, however, that within some characteristic range of densities $[\rho_1, \rho_2]$, where $\rho_1 < \rho_{max} < \rho_2$, $j(\rho_1) > j(\rho_2)$, two long-lived traffic states with a different flux can be reached, depending on initial conditions. Therefore the flux j within this range will show hysteresis. Our arguments for the first-order phase transition are still valid for the branch where $j(\rho^+)$ at the phase transition is analytic, i.e. $\rho^+ > \rho_2$. On the contrary, on the branch where $\rho^- > \rho^-_{crit}$, then $j(\rho^-_{crit}) = j(\rho_2)$, and the exact phase transition line will disperse and, further, high-velocity fluctuations due to the hysteresis will be observed. Both regimes were observed experimentally on Korean highways ([18], phases CT5 and CT4). Our empirical traffic data evidently correspond to the branch without (or with small) hysteresis, because the universal outflow is seen (figure 4).

Finally, we remark that the scope of the domain-wall theory goes far beyond the correct prediction of the stationary phase diagram. Recent work on the asymmetric simple exclusion process has shown that even the dynamics of the shock position [19], and therefore also fluctuation quantities, are matched by the domain-wall theory [20] and are applicable to very small systems. Also the results for the interplay between different stationary states obtained by the domain-wall theory are in excellent agreement with simulation results [20]. This shows that the domain-wall theory offers a quite complete description of the observables which play a major role in analysing traffic networks. Another advantage of the theory is that it can be applied to a large class of traffic models because the only prerequisite is a single-valued fundamental diagram (see [12] for a more sophisticated example). However, also in the presence of hysteresis, similar types of arguments may be applied in order to establish the phase diagram of open systems [21].

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